

Angular Tunneling Effect

Cássio Lima¹, Jorge Henrique Sales¹, A. T. Suzuki².

¹*Universidade Estadual de Santa Cruz, Departamento de Ciências Exatas e Tecnológicas, 45662-000 - Ilhéus, BA, Brasil and*

²*Instituto de Física Teórica, Universidade Estadual Paulista*

Rua Dr. Bento Teobaldo Ferraz, 271 - 01140-070 — São Paulo, SP - Brazil.

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We investigate the tunneling of an electron with momentum p in the direction of V potential and under an angle θ to the normal potential. Using the boundary conditions, the conditions of continuity and Snell's law, we obtain tunneling for various angles.

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I. INTRODUCTION

The alpha-decay process was interpreted in the early 1920s in terms of tunnelling through a quantum mechanical potential barrier [1, 2]. Nuclear deformation effects have been studied in 1950s by Bohr et al [3] and Froman [4]. Recently, two theoretical extreme approaches have been developed to describe the alpha decay: the cluster- and sion-like theories [5]. A lot of new experimental and theoretical investigation on alpha decay half-life has been developed during the last three years or so [6, 7,8,10,11,12,13,14,15,16,17,18,19]. In addition, half-life values for spontaneous nuclear decay processes (proton emission, alpha decay, cluster radioactivity, and cold sion) have been presented very recently in the framework of the Effective Liquid Drop Model [20].

Penetration property in a classically forbidden region allows the understanding of several phenomena such as electron tunneling and alpha decay, from the analysis of the behavior of a particle concerning a potential barrier rectangular figure 1, where it can be transmitted and reflected.

A potential barrier is defined by

$$V(x) = \begin{cases} V & \text{se } 0 \leq x \leq a \\ 0 & \text{se } x < 0 \text{ ou } x > a \end{cases} \quad (1)$$

According to classical physics, a particle of energy E less than the height V of a barrier could not penetrate – the region inside the barrier is classically forbidden. But the wavefunction associated with a free particle must be continuous at the barrier and will show an exponential decay inside the barrier. The wavefunction must also be continuous on the far side of the barrier, so there is a finite probability that the particle will tunnel through the barrier. The transmission coefficient T is the probability of a particle incident from the left (region 1) to be tunneling through the barrier (region 2) and continue to travel to the right (region 3) is given by

$$\mathfrak{T} \simeq 16 \frac{E}{V} \left(1 - \frac{E}{V}\right) e^{-2k_2 a}. \quad (2)$$

Equation (2) shows us that a particule with mass m and energy $E < V$, that approaches a potencial barrier V and a wide has a probability $\mathfrak{T} \neq 0$ of penetrated the barrier and appear on the other side. This phenomena is known as tunneling.

II. ANGULAR TUNNELING

The model presented in this section serves for scattering, collimated and nearly monoenergetic beam of particles, dispersions that wave packets are very small, and the reflection and transmission coefficients can be determined from the components of the propagation of monochromatic plane wave eigenfunctions the Hamiltonian of the free particle, with free energy E . Thus, if the particle of mass m approaches the barrier around the region 1(Fig.1) with incidence angle θ_1 with normal line to the barrier surface, the incident and reflected state is represented by the wave $\psi_1(r)$

$$\psi_1(r) = Ae^{i\vec{k}_1 \cdot \vec{r}} + Be^{-i\vec{k}_1 \cdot \vec{r}} \quad (3)$$

where $\vec{k}_1 \cdot \vec{r} = k_1 r \cos \theta_1$, being the vector \vec{r} any position in the plane that separates two regions 1 and 2 Fig.?? e $\vec{k}_1 = \frac{\vec{p}_1}{\hbar}$. The second installment of the wave function is the state associated with the reflection of the particle through the barrier.

The wave function which crosses the barrier is given by

$$\psi_2(r) = Ce^{i\vec{k} \cdot \vec{r}} + De^{-i\vec{k} \cdot \vec{r}}, \quad \vec{k} \cdot \vec{r} = kr \cos \theta_2 \quad (4)$$

and the transmitted wave function, passing the region 2 to region 3 is given by

$$\psi_3(r) = Ee^{i\vec{k}_1 \cdot \vec{r}}, \quad \vec{k}_1 \cdot \vec{r} = k_1 r \cos \theta_3 \quad (5)$$

Assuming that the absolute refractive index for region 1 is $n_1 = \frac{c}{v_1}$ and it is equal to the region 3, i.e., $n_1 = n_3$, this implies by Snell's refraction equation $\theta_3 = \theta_1$. So, for region 3 the emerging wave function from barrier 3 $\psi_3(r)$ depends on θ_1 incidence angle.

Thus, for region 3, the emerging wave function from barrier $\psi_3(r)$ depends on θ_1 incidence angle.

First of all lets compute for the case of the step potential, $E > V$, and we need to treat the problem using the boundary conditions defined at $r = 0$ and $r = a$. The current probability must remain continuous at the origin, despite the discontinuity of potential. This implies that the wave function and its first derivative with respect to r should be continuous in the case $r = 0$

$$\Psi_1(0) = \Psi_2(0) \implies A + B = C + D \quad (6)$$

The first derivative of the wave functions are

$$\begin{aligned} \frac{\partial \Psi_1}{\partial r} &= ik_1 r \cos \theta_1 \left(Ae^{i\vec{k}_1 \cdot \vec{r}} + Be^{-i\vec{k}_1 \cdot \vec{r}} \right) \\ \frac{\partial \Psi_2}{\partial r} &= ikr \cos \theta_2 \left(Ce^{i\vec{k} \cdot \vec{r}} - De^{-i\vec{k} \cdot \vec{r}} \right) \\ \frac{\partial \Psi_3}{\partial r} &= ik_1 r \cos \theta_1 \left(Ee^{i\vec{k}_1 \cdot \vec{r}} \right) \end{aligned} \quad (7)$$

for $r = 0$, we have

$$\begin{aligned} ik_1 \cos \theta_1 (A - B) &= ik_2 \cos \theta_2 (C - D) \\ A - B &= \frac{1}{n} \frac{\cos \theta_2}{\cos \theta_1} (C - D) \end{aligned}$$

where

$$n = \frac{k_1}{k_2}$$

In the case $r = a$

$$Z_1 C + \frac{1}{Z_2} D = Z_1 E$$

For the first derivative at $r = a$

$$\begin{aligned}
ik_2 \cos \theta_2 CZ_2 - \frac{ik_2 \cos \theta_2}{Z_2} D &= iEZ_1 k_1 \cos \theta_1 \\
CZ_2 - \frac{1}{Z_2} D &= Z_1 E n \frac{\cos \theta_1}{\cos \theta_2}
\end{aligned}$$

and

$$\begin{aligned}
Z_1 &= e^{ik_1 a \cos \theta_1} \\
Z_2 &= e^{ik_2 a \cos \theta_2}
\end{aligned}$$

Solving the equation systems, we have

$$A = \frac{1}{4N} \frac{Z_1}{Z_2} (1+N)^2 \left[1 - \frac{(N-1)^2}{(N+1)^2} \cdot Z_2^2 \right] E$$

where $N = n\alpha$ and $\alpha = \frac{\cos \theta_1}{\cos \theta_2}$. In the limit of $\theta_1 = \theta_2 = 0$, N tends to the refractive index n .
The calculation of the amplitude for the barrier is given by:

$$T = \frac{E}{A}$$

$$T = \frac{4NZ_2}{Z_1} \frac{1}{(N+1)^2} \frac{1}{\left[1 - \left(\frac{N-1}{N+1} \right)^2 Z_2^2 \right]}$$

or

$$T = \frac{4N}{(1+N)^2} \frac{e^{i(k_2-k_1)a}}{\left[1 - \left(\frac{N-1}{N+1} \right)^2 \right] e^{2ik_2 a}} \quad (8)$$

In the case for tunneling: $0 < E < V$

$$V(x) = \begin{cases} 0, & x < 0 \rightarrow \text{region 1} \\ V > 0, & 0 < x < a \rightarrow \text{region 2} \\ 0, & x > a \rightarrow \text{region 3} \end{cases}$$

The above solutions are still valid, now we just take

$$k_2 = i |k_2| = \frac{i}{\hbar} \sqrt{2m(V-E)} \equiv iK, \quad k > 0$$

The wave equation for region 2 is

$$\psi_2(r) = Ce^{-kr} + De^{kr} \quad (0 \leq r \leq a)$$

but here, unlike the step, one can not exclude the exponentially increasing because r does not extend to $-\infty$ (boundary region) because we have a limit to the potential that is up to $r = a$. Another important fact is that the index of refraction becomes imaginary, i.e.

$$n = \frac{k_1}{k_2} = i \frac{k_1}{k} = -i\eta \quad (\eta > 0)$$

thus N is

$$N = i \sqrt{\left(\frac{\eta^2 + \sin^2 \theta_1}{1 - \sin^2 \theta_1} \right)}$$

or we can define N as

$$N = i\beta$$

where

$$\beta = \sqrt{\left(\frac{\eta^2 + \sin^2 \theta_1}{1 - \sin^2 \theta_1} \right)}$$

Substituting into Eq.(8)

$$T = \left[\frac{\frac{4i\beta}{(i\beta+1)^2}}{1 - \left(\frac{i\beta-1}{i\beta+1} \right)^2 e^{-2Ka}} \right] e^{-Ka - ik_1 a}$$

where

$$K = \frac{\sqrt{2m(V-E)}}{\hbar}$$

For a thick barrier $ka \gg 1$ implies despise e^{-2ka} in the denominator, then:

$$\Im = |T|^2 = \frac{16\beta^2}{(\beta^2 + 1)^2} e^{-2ka}$$

As

$$\begin{aligned} \eta &= \frac{k_1}{K} = \frac{\frac{i}{\hbar} \sqrt{2mE}}{\frac{i}{\hbar} \sqrt{2m(V-E)}} \\ \eta &= \sqrt{\frac{E}{(V-E)}} \rightarrow \eta^2 = \frac{E}{(V-E)} \end{aligned}$$

and

$$\beta^2 = \frac{E(1 - \sin^2 \theta)V \sin^2 \theta}{(V - E)(1 - \sin^2 \theta_1)}$$

$$\beta^2 + 1 = \frac{V}{(V - E)(1 - \sin^2 \theta_1)}$$

Finally we obtain

$$\mathfrak{T}_{\text{angular}} = \mathfrak{T} - \mathfrak{T} \sin^2 \theta_1 + \frac{8}{V^2} (V - E)^2 \sin^2(2\theta_1) e^{-2ka} \quad (9)$$

Eq.(9) indicates two terms more on the equation and tunneling \mathfrak{T} where we observe the sine function present. Thus, for $\theta = 0^\circ$ we obtain the usual tunneling. The following graphs compare the usual tunneling with the angular tunneling. The x-axis represents the particule energy, whereas y-axis the tunneling probability. As we can see, there is a range (between 30° and 45°) where the angular tunneling is more favorable than the usual tunneling for energy below to 6 eV (Fig. 2). For $\theta = 0^\circ$ we obtain the usual tunneling, i.e., the angular tunneling is equal to the usual tunneling. For $\theta = 90^\circ$ there is no angular tunneling since the particle finds no obstacle Fig.2. It is important to highlight that our results were obtained for a electron ranging from 1 eV to 12 eV, striking a potencial barrier with 12 eV and 0.18 nm wide.

III. CONCLUSION

In this paper we have demonstrated that the incident angle influences the probability of tunneling. According to our results there is a range (between 30° and 45°) where the angular tunneling is more favorable than the usual tunneling for energy below to 6 eV. For $\theta = 0^\circ$ we obtain the usual tunneling, i.e., the angular tunneling is equal to the usual tunneling. For $\theta = 90^\circ$ there is no angular tunneling since the particle finds no obstacle.

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